Chain Rule Challenges

Differentiate the following functions.

1. $w(x) = \sin\left(\tan\left(\sqrt{1+\cos(x)}\right)\right)$

2. $y = \sin\left(\sqrt{e^{2x^3+5x-4}}\right)$

3. $G(x) = [x\sin(2x) + (\tan(x^7))^4]^5$

4. $F(x) = \dfrac{\cos(2x+3)}{e^{x^3-5x+4}}$

Solutions

1. $w(x) = \sin\left(\tan\left(\sqrt{1+\cos(x)}\right)\right)$

$$w'(x) = \cos\left(\tan\left(\sqrt{1+\cos(x)}\right)\right)\cdot\frac{d}{dx}\left(\tan\left(\sqrt{1+\cos(x)}\right)\right)$$

$$= \cos\left(\tan\left(\sqrt{1+\cos(x)}\right)\right)\cdot\sec^2\left(\sqrt{1+\cos(x)}\right)\frac{d}{dx}\left(\sqrt{1+\cos(x)}\right)$$

$$= \cos\left(\tan\left(\sqrt{1+\cos(x)}\right)\right)\cdot\sec^2\left(\sqrt{1+\cos(x)}\right)\cdot\frac{1}{2\sqrt{1+\cos(x)}}\cdot\frac{d}{dx}\left(1+\cos(x)\right)$$

$$= \cos\left(\tan\left(\sqrt{1+\cos(x)}\right)\right)\cdot\sec^2\left(\sqrt{1+\cos(x)}\right)\cdot\frac{1}{2\sqrt{1+\cos(x)}}\cdot\left(-\sin(x)\right)$$

$$= -\frac{\sin(x)\cos\left(\tan\left(\sqrt{1+\cos(x)}\right)\right)\sec^2\left(\sqrt{1+\cos(x)}\right)}{2\sqrt{1+\cos(x)}}$$

2. $y = \sin\left(\sqrt{e^{2x^3+5x-4}}\right)$

$$\frac{dy}{dx} = \cos\left(\sqrt{e^{2x^3+5x-4}}\right)\cdot\frac{d}{dx}\left(\sqrt{e^{2x^3+5x-4}}\right)$$

$$= \cos\left(\sqrt{e^{2x^3+5x-4}}\right)\cdot\frac{1}{2\sqrt{e^{2x^3+5x-4}}}\cdot\frac{d}{dx}\left(e^{2x^3+5x-4}\right)$$

$$= \cos\left(\sqrt{e^{2x^3+5x-4}}\right)\cdot\frac{1}{2\sqrt{e^{2x^3+5x-4}}}\cdot e^{2x^3+5x-4}\cdot\frac{d}{dx}\left(2x^3+5x-4\right)$$

$$= \cos\left(\sqrt{e^{2x^3+5x-4}}\right)\cdot\frac{1}{2\sqrt{e^{2x^3+5x-4}}}\cdot e^{2x^3+5x-4}\cdot\left(6x^2+5\right)$$

$$= \frac{\left(6x^2+5\right)e^{2x^3+5x-4}\cos\left(\sqrt{e^{2x^3+5x-4}}\right)}{2\sqrt{e^{2x^3+5x-4}}}$$

$$= \frac{\left(6x^2+5\right)\sqrt{e^{2x^3+5x-4}}\cos\left(\sqrt{e^{2x^3+5x-4}}\right)}{2}$$

3. $G(x) = [x\sin(2x) + (\tan(x^7))^4]^5$

$$G'(x) = 5[x\sin(2x)+(\tan(x^7))^4]^4\cdot\frac{d}{dx}\left(x\sin(2x)+(\tan(x^7))^4\right)$$

$$= 5[x\sin(2x)+(\tan(x^7))^4]^4\cdot\left[\frac{d}{dx}(x)\cdot\sin(2x)+x\cdot\frac{d}{dx}\left(\sin(2x)\right)+4\left(\tan\left(x^7\right)\right)^3\cdot\frac{d}{dx}\left(\tan\left(x^7\right)\right)\right]$$

$$= 5[x\sin(2x)+(\tan(x^7))^4]^4\cdot\left[1\cdot\sin(2x)+x\cdot\cos(2x)\frac{d}{dx}(2x)+4\left(\tan\left(x^7\right)\right)^3\cdot\sec^2\left(x^7\right)\cdot\frac{d}{dx}\left(x^7\right)\right]$$

$$= 5[x\sin(2x)+(\tan(x^7))^4]^4\cdot\left[\sin(2x)+x\cdot\cos(2x)\cdot 2+4\left(\tan\left(x^7\right)\right)^3\cdot\sec^2\left(x^7\right)\cdot 7x^6\right]$$

$$= 5[x\sin(2x)+(\tan(x^7))^4]^4\cdot\left[\sin(2x)+2x\cos(2x)+28x^6\sec^2\left(x^7\right)\left(\tan\left(x^7\right)\right)^3\right]$$

4. $F(x) = \dfrac{\cos(2x+3)}{e^{x^3-5x+4}}$

$$F'(x)=\frac{e^{x^3-5x+4}\cdot\frac{d}{dx}\left(\cos(2x+3)\right)-\cos(2x+3)\cdot\frac{d}{dx}\left(e^{x^3-5x+4}\right)}{\left(e^{x^3-5x+4}\right)^2}$$

$$=\frac{e^{x^3-5x+4}\cdot\left(-\sin(2x+3)\right)\cdot\frac{d}{dx}\left(2x+3\right)-\cos(2x+3)\cdot e^{x^3-5x+4}\cdot\frac{d}{dx}\left(x^3-5x+4\right)}{\left(e^{x^3-5x+4}\right)^2}$$

$$=\frac{e^{x^3-5x+4}\cdot\left(-\sin(2x+3)\right)\cdot 2-\cos(2x+3)\cdot e^{x^3-5x+4}\cdot\left(3x^2-5\right)}{\left(e^{x^3-5x+4}\right)^2}$$

$$=\frac{e^{x^3-5x+4}\left(-2\sin\left(2x+3\right)-\left(3x^2-5\right)\cos(2x+3)\right)}{\left(e^{x^3-5x+4}\right)^2}$$

$$=\frac{-2\sin\left(2x+3\right)-\left(3x^2-5\right)\cos(2x+3)}{e^{x^3-5x+4}}$$

$$=\frac{-2\sin\left(2x+3\right)-3x^2\cos\left(2x+3\right)+5\cos\left(2x+3\right)}{e^{x^3-5x+4}}$$