

# Introduction to the Practice of Statistics

Fifth Edition  
Moore, McCabe

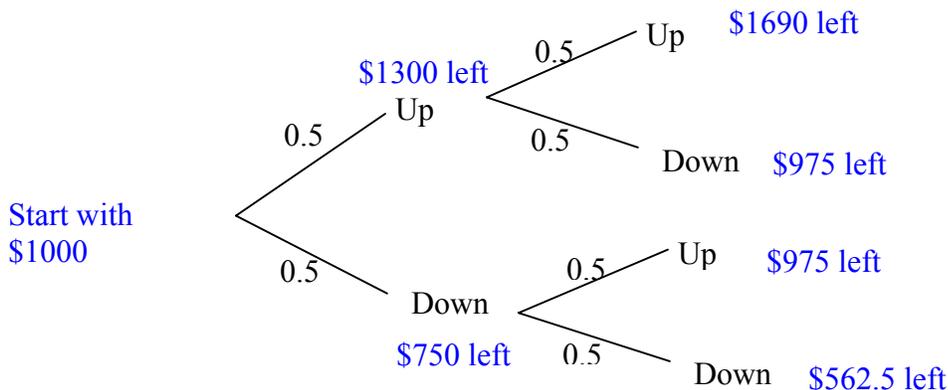
## Section 4.4 Homework

4.65 You buy a hot stock for \$1000. The stock either gains 30% or loses 25% each day, each with probability 0.5. Its returns on consecutive days are independent of each other. You plan to sell stocks after two days.

- a) What are the possible values of the stock two days, and what is the probability for each value? What is the probability that the stock is worth more after two days than the \$1000 you paid for it?

$$P(\text{Stock goes up}) = 0.5 \quad P(\text{Stock goes down}) = 0.5, \quad P(\text{Stock goes up} \mid \text{stock goes up}) = 0.5$$

Let us use a tree diagram to help us see the situation better.



I can now see that there are only three possible values. I can use the tree diagram to figure out the probabilities. Let the random variable  $X$  equal the amount of money left after two days.

$X$	1690	975	562.5
$P(X)$	.25	.5	.25

To answer the second question in terms of the random variable  $X$ , I want  $P(X = 1690) = 0.25$

- b) What is the mean value of the stock after two days? You see that these two criteria give different answers to the question, “Should I invest?”

$$\mu = 1690(0.25) + 975(0.5) + 562.5(0.25)$$

= \$1050.63 This value will appear whenever we consider a long run view. So in the long run, for a two day investment under the mentioned assumptions, we would expect to earn \$50.63 on average.

4.67 The rules for means and variances allow you to find the mean and variance of a sum of random variables without first finding the distribution of the sum, which is usually much harder to do.

This problem is taking you step by step to show why we would want to use the rules of means and standard deviations.

- a) A single toss of a balanced coin has either 0 or 1 head, each with probability of  $\frac{1}{2}$ . What are the mean and standard deviation of the number of heads?

Let us define the random variable Y to be the number of heads that appear after one toss.

Thus the discrete table associated with one coin toss is

Y	0	1
P(Y)	0.5	0.5

The mean of this distribution is  $\mu = 0(0.5) + 1(0.5) = 0.5$

The variance is given by  $\sigma^2 = (0 - 0.5)^2(0.5) + (1 - 0.5)^2(0.5) = 0.25$

So the standard deviation is  $\sigma = 0.5$

- b) Toss a coin four times. Use the rules for means and variances to find the mean and standard deviation of the total number of heads.

Let the random variable T be the number of heads that can appear out of four tosses:  $T: \{0, 1, 2, 3, 4\}$ .

The rule you want to use here  $\mu_{x+y} = \mu_x + \mu_y$  (Note : we can repeat the process for a finite number of distributions), **use rule 2 found on page 298 (see example 4.24).**

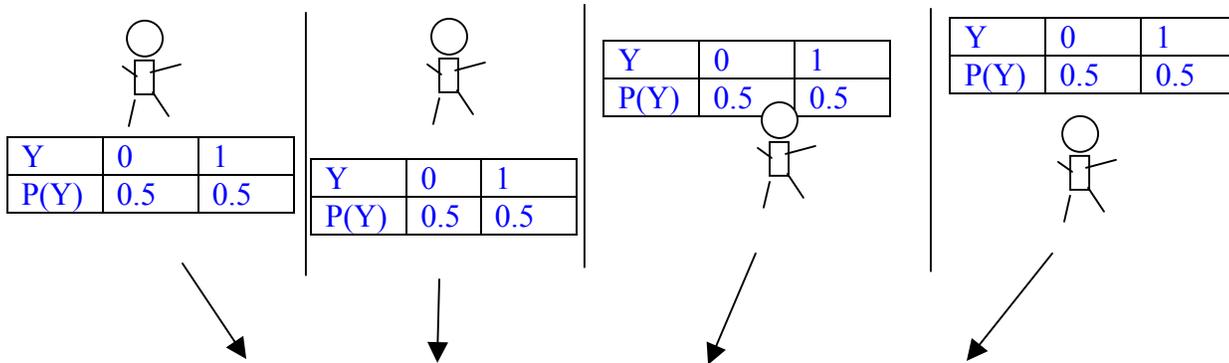
**Thus, without much explanation, what we want is**

$\mu_T = \mu_{Y+Y+Y+Y} = \mu_Y + \mu_Y + \mu_Y + \mu_Y$ , **that is combine the mean four times.**

$\mu_T = 0.5 + 0.5 + 0.5 + 0.5 = 2$

**Why? Below is an explanation that seemed to work with one student.**

Obviously we could have a person toss a coin four times. But suppose that there are four people tossing one coin once. We then pool the result from each person to create one instance in the four toss case. As far as each individual is concerned the only table that applies to them is the one found in part (a).



Collect each individual result so we can create the new distribution. The mean of this new distribution will be 2,  $\mu_T = 2$ . But each individual person is dealing with a distribution with a mean of 0.5,  $\mu_Y = 0.5$ .

$$\begin{aligned} \sigma_T^2 &= \sigma_Y^2 + \sigma_Y^2 + \sigma_Y^2 + \sigma_Y^2 \\ &= 0.5^2 + 0.5^2 + 0.5^2 + 0.5^2 \\ &= 1 \end{aligned}$$

- c) Example 4.17 (page 280) finds the distribution of the number of heads in four tosses. Find the mean and standard deviation from this distribution. Your results in (b) and (c) should agree.

T	0	1	2	3	4
P(T)	0.0625	0.25	0.375	0.25	0.0625

$$\begin{aligned} \mu_T &= 0(0.0625) + 1(0.25) + 2(0.375) + 4(0.0625) \\ &= 2 \end{aligned}$$

$$\begin{aligned} \sigma_T^2 &= (0 - 2)^2(0.0625) + (1 - 2)^2(0.25) + (2 - 2)^2(0.375) + (3 - 2)^2(0.25) + (4 - 2)^2(0.0625) \\ &= 1 \\ \sigma_T &= 1 \end{aligned}$$

**Insurance.** *This business of selling insurance is based on probability and the law of large numbers. Consumers buy insurance because we all face risks that are unlikely but carry high cost. Think of a fire destroying your home. So we form a group to share the risk; we all pay a small amount, and the insurance policy pays a large amount to those few of us whose homes burn down. The insurance company sells many policies, so it can rely on the law of large numbers. Exercises 4.79 to 4.82*

4.79 An insurance company looks at the records for millions of homeowners and sees that the mean loss from fire in a year is  $\mu = \$250$  per person. Most of us have no loss, but a few lose their homes. The  $\mu = \$250$  is the average loss. The company plans to sell fire insurance for \$250 plus enough cover its costs and profit. Explain clearly why it would be stupid to sell only 12 policies. Then explain why selling thousands of such policies is a safe business.

Let us consider people who did have a fire in their home, and calculated the mean loss of these people. Let us say that this mean loss is \$40,000 on average. We assume that fire is a rare event. Say that I am able to sell to 12 people fire insurance for \$1000 a year. The most likely scenario, is that I will collect \$12,000 every year since fire is a rare event. Let us suppose that I go ten years without a mishap. In those ten years I would only collect \$120,000 (I have not calculated the interest I could earn if I invested this money). Let us say that in the tenth year the unthinkable happens one person has a fire that destroys the home for a loss of \$150,000. I would not be able to cover the loss with the money I have collected. While the risk is unlikely, should the unlikely happen the loss could be much higher than the reward; the  $\mu = \$250$  stated above does not apply for 12 policies.

By selling thousands of policies, however, I now face the real risk that every year at least one customer will have a loss due to a fire. Since I am selling thousands of policies, say the high thousands, the  $\mu = \$250$  parameter begins to apply. Let us say I sell 50,000 policies at \$500 a policy per year. I would collect \$5million dollars a year. If I have 20 people having fire damage a year averaging \$100,000 each I would be able to cover that loss and still make a profit. The question is how rare is a fire.

Using the  $\mu = \$250$  as an exact number I see that the I would expect to cover \$2.5 million dollars a year in fire damage;  $50,000(\$250)$ . Again, I would have enough money to cover costs of the business.

4.81 According to the current Commissioners' Standard Ordinary mortality table, adopted by state insurance regulators in December 2002, a 25-year old man has the probabilities of dying during the next five years.

Age at death	25	26	27	28	29	> 29
Probability	0.00039	0.00044	0.00051	0.00057	0.00060	0.99236

a) What is the probability that the man does not die in the next five years?

$$\begin{aligned}
 P(> 29) &= 1 - P(\text{dies between 25}^{\text{th}} \text{ and 29}^{\text{th}} \text{ birthday}) \\
 &= 1 - (0.00039 + 0.00044 + 0.00051 + 0.00057 + 0.00060) \\
 &= 0.99236
 \end{aligned}$$

b) An online insurance site offers a term insurance policy that will pay \$100,000 if a 25-year old man dies within the next 5 years. The cost is \$175 per year. So the insurance company will take in \$875 from this policy if the man does not die within five years. If he does die, the company must pay \$100,000. Its loss depends on how many premiums were paid, as follows:

Age at death	25	26	27	28	29	> 29
Loss	\$99,825	\$99,650	\$99,475	\$99,300	\$99,125	

What is the insurance company's mean cash intake from such policies?

Loss	-\$99,825	-\$99,650	-\$99,475	-\$99,300	-\$99,125	\$875
Probability	0.00039	0.00044	0.00051	0.00057	0.00060	0.99236

$$\begin{aligned}
 \mu &= -\$99,825(0.00039) + -\$99,650(0.00044) + -\$99,475(0.00051) + -\$99,125(0.00060) + \$875(0.99236) \\
 &= \$618.73
 \end{aligned}$$